Available online at www.sciencedirect.com
SciVerse ScienceDirect
Procedia Engineering 34 (2012) 688 – 693
Procedia Engineering
www.elsevier.com/locate/procedia

9th Conference of the International Sports Engineering Association (ISEA)

Using experimental modal analysis to validate a finite element model of a tennis racket

Guy Banwell[a]*, Stefan Mohr[b], Steve Rothberg[a], Jon Roberts[a]

[a]Sports Technology Institute, Loughborough University, 1 Oakwood Drive, Loughborough, LE11 3QF
[b]Head Sport GmbH Wuhrkopfweg 1, A_6921 Kennelbach, Austria

Accepted 06 March 2012

Abstract

A finite element (FE) model of a tennis racket was created to aid in the design and development of new rackets by allowing engineers to analyse the mechanical properties of the frames without making physical prototypes. This approach saves both time and money but only if the model is an accurate representation of the manufactured racket. The FE model, therefore, needed to be validated. This paper presents a method of dynamically validating the FE model by comparing experimental modal analysis (EMA) data measured from a manufactured racket with the mode shapes calculated by the finite element analysis software. A mechanical shaker was used to excite the racket and a scanning laser Doppler vibrometer (SLDV) measured the response of the racket in three axes of motion. The experimental setup acquired out-of-plane (normal to the plane of the stringbed) bending, torsional and string-bed mode shapes that have been reported in past literature. In addition, in-plane (parallel to the stringbed plane) bending modes were also excited and measured, which have not been reported before. Comparison of the experimental and theoretical data revealed that the natural frequencies and corresponding mode shapes correlate well between the manufactured racket and the FE model, therefore validating the model and the method used to construct the model.

Keywords: Tennis; modal analysis; finite element analysis; validation

1. Introduction

Computer modelling, usually Finite Element Analysis (FEA), has been extensively used within the sporting goods industry to aid in the design process of new products. FE modelling has been shown to be

* *Guy Banwell. Tel:* +44 (0)1509 564807.
E-mail: G.Banwell@lboro.ac.uk.

doi:10.1016/j.proeng.2012.04.117

capable of predicting the required mechanical properties of a tennis racket that will give the desired performance characteristics. Researchers [1,2] have developed computer models to study the effect of stringbed tension on racket deflection and coefficient of restitution respectively, but the FE data were not compared with experimental results. Allen et al [3] used an FE model to investigate the influence of frame and stringbed properties on the rebound characteristics of the ball; they compared the simulated data to experimental data from a similar racket to validate the model.

FE modelling has been shown to be a useful tool in optimizing performance characteristics; it is also a useful technique that can be used to reduce the time and financial cost of designing and manufacturing a new tennis racket. Before a new design of a tennis racket can be manufactured on the production line, the engineers must first be satisfied that the end product will have the desired mechanical properties. Before the advent of computer modelling, the traditional method was to manufacture prototypes, which were then subjected to mechanical tests to measure the structure's properties. This process would have to be repeated until a prototype satisfied all the test requirements. The length of this process is dependent very much on the skill and experience of the engineer and can be very costly and time consuming for the company. An accurate FE model can be used to predict the mechanical properties of the manufactured racket without the need for numerous prototypes.

To ensure that an FE model is accurate it needs to be validated. Modal analysis can be used as a method to dynamically validate the FE model by comparing its first few modes of vibration with experimental modal analysis data from the matching manufactured part [4]. Hocknell et al [5] compared the frequencies of the modes calculated from an FE model and those measured from the corresponding golf club head; the model was assumed to be valid as the natural frequencies of the modes closely correlated. Although the mode shapes were also visually compared to assess their similarity it was not possible to quantify the similarity. Since Hocknell et al [5] completed their study, advances in computer modelling have allowed experimental and simulated modal data to be analysed side by side for comparison.

The primary aim of this paper was to develop a method capable of determining the accuracy, and hence validity, of a computational FE model by comparing its modal properties to the manufactured tennis racket that it embodies. Based on the validity of the FE model, the techniques used to construct the FE model will be evaluated to determine the suitability of using such techniques.

Although the majority of the energy excited in a racket from a tennis ball impact is concentrated below 200 Hz due to the contact time, previous research [6] has shown that frequencies up to 1500 Hz are present in the frame of a tennis racket. For this reason it was decided that the FE model should be an accurate representation of the dynamic properties of the tennis racket up to 1500 Hz.

2. Finite Element Model Development

Head Sport GmbH use finite element modelling to aid in the design process of their tennis rackets. The FE models of tennis racket can have very different levels of complexity depending on the modelling techniques used, this paper will examine if a relatively simple FE model can adequately represent the mechanical properties of the manufactured racket. A finite element analysis software package (ANSYS) was used to construct a model of a HEAD AirFlow 7 tennis racket. The frame was modelled as an isotropic material with an e-modulus of 53.5 GPa. In reality, the racket frame is constructed from carbon fibre, which has orthotropic properties. The strings were modelled on a single plane with the main and cross strings bonded at the intersections rather than interwoven. The material of the strings was PET with a literature-defined Young's modulus of 2.2 GPa. The diameter and density of the strings was adjusted to match the total stringbed mass of HEAD Sonic Pro strings strung with a pretension of 245 N (55 lbs).

The modelling of the strings was simplified further by defining the PET as having linear elastic properties.

Altering the temperature of the model was used to adjust the tension of the strings of the FE model. As the string material had a thermal expansion coefficient while the frame material did not, only the mechanical stress of the strings was increased when virtually cooling the model down. With these material parameters defined, modal analysis of the racket was conducted within the ANSYS software. In total 100 natural frequencies and corresponding mode shapes were calculated from 180 Hz to 3188 Hz. The modal density of the FE racket is far higher as the frequencies exceed 1500 Hz, so only the 16 modes below 1500 Hz will be compared to the experimental data.

3. Experimental modal analysis

As the experimental modal analysis data were to be used to create the reference model, to which the FE model would be compared, it was crucial that these data were as accurate as possible. The discretised geometry of the racket was defined with small circular markers placed on the racket, which enabled a simple but accurate two-dimensional wire-frame model to be created using an optical measurement technique (GOM Triptop). Figure 1 illustrates the creation and orientation of the wireframe model with the global axis system shown. The circular markers were directly used as the measurement points thereby removing any discrepancy between the position of the points in the model and the actual measurement points. 79 markers were placed around the frame and on intersections of the stringbed as shown in Figure 1.

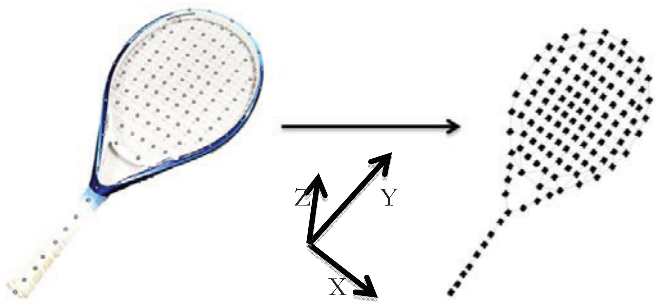

Fig. 1. Tennis racket with optical markers and the resultant wireframe model with global axis

Initially the racket was excited with a modally tuned impact hammer and the response measured with a lightweight accelerometer using the roving excitation technique. It became apparent, however, that the type of impact hammer (B&K 8203) used was not capable of providing adequate excitation up to 1500 Hz when impacting the stringbed. This was due to the longer contact time between the strings and the impact hammer when compared to that with the frame and the impact hammer. To ensure a sufficient excitation frequency range, an electromagnetic shaker was used to excite the racket. The response of the racket was measured using a scanning laser Doppler vibrometer, rather than an accelerometer, which would have added mass to the racket. Modal data from the initial investigation with the impact hammer were used to select an appropriate point for the shaker to be attached so that all the desired modes were excited. This initial set of data was also compared to the modal data acquired with the shaker to ensure that the attachment of the shaker did not add any additional mass to the system.

The electromagnetic shaker was configured such that it exerted a forcing function normal to the plane of the stringbed (z-axis in Figure 1) on the frame at the location where the yoke joins the frame. The aluminium frame, from which the racket was freely suspended with nylon threads, was designed in such a way that allowed the racket to be suspended both vertically and horizontally so that the response of the racket could be measured in all three axes without changing the location of the vibrometer. Although the response of the racket frame was measured both in-plane and out-of-plane, the excitation force was always normal to the stringbed (out-of-plane). Preliminary testing revealed that in-plane modes were excited by this method.

The racket was excited with a burst random signal (white noise) with a bandwidth of 3.2 kHz and an excitation time of 50% of the total measurement duration of 0.32 seconds. As well as ensuring that the vibrations in the racket had decayed entirely before the end of the measurement run to avoid leakage errors, this method also meant that the transient response of the racket was measured. The recorded data had a spectral resolution of 3.125 Hz and were presented as the means of 20 repeated measurement runs.

On average, the level of the FRFs measured out-of-plane were approximately an order of magnitude greater than those measured in-plane (Figure 2), this is because the racket was only excited in an out-of-plane direction. In order to view the in-plane measurements and mode shapes the out-of-plane velocity component had to be disabled for these particular modes. In total 16 natural frequencies of the racket were detected below 1500 Hz and the corresponding mode shapes were calculated within the software. Figure 3b illustrates the first bending mode of the racket (B1).

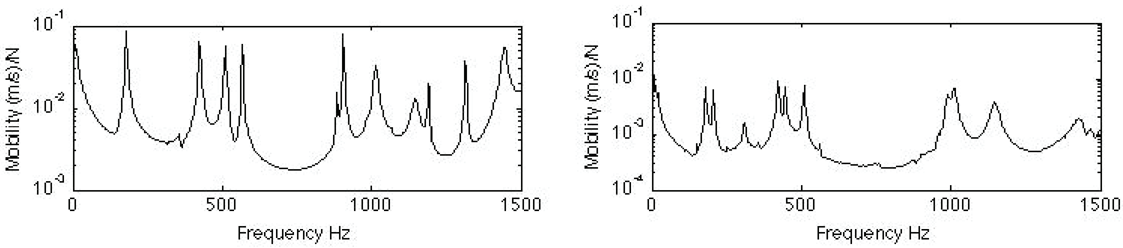

Fig. 2. Sum of FRFs measured out-of-plane (a) and in-plane; (b) from AirFlow 7 strung with Sonic Pro strings at 245 N (55 lbs.)

4. Comparison and Validation of the FE model

Once the EMA data had been collected, the temperature of the FE model was altered until the frequency of the first stringbed mode matched that of the experimental data. A full set of mode shapes was then exported from the FE model to be compared with the reference data. Table 1 details the comparison of the modes in terms of their frequencies with the difference displayed as a percentage. The modes are labelled with a **B**, **IP B**, **T** or **S** referring to whether they are considered (Out-of-Plane) **B**ending, **I**n-**P**lane **B**ending, **T**orsion or **S**tringbed respectively. The finite element model's prediction of natural frequency are within 2 % of the experimental data values apart from the in-plane modes, which differ by as much as 33.92 % for IP B3.

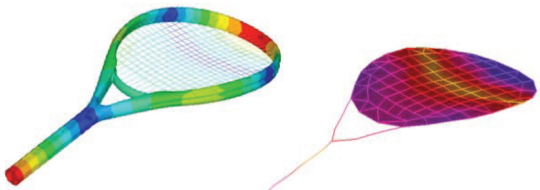

Fig. 3. Mode shape of B1 from (a) FE model; (b) experimental modal analysis

While Table 1 compares the natural frequencies of the modal data it does not indicate how well the mode shapes correlate. A result file was exported from ANSYS that included the FE model as well as the accompanying modal data; this file was then imported into LMS Virtual.lab along with the experimental data and wireframe model from LMS Test.lab. The FE model was transformed from the 3D mesh geometry into a wireframe model with 79 nodes that aligned with the reference geometry. After the modal data of the verification model had been transformed to match the reduced geometry, the two sets of data were compared. A modal assurance criterion (MAC) was used to calculate the degree of relationship between the vectors of the modes within the experimental and the FE model. Figure 4 illustrates the MAC correlation of the experimental and the finite element analysis modal analysis, a value of 1 indicates that the mode shapes are identical where as a value of 0 reveals that there is no correlation between the mode shapes. Two separate analyses were performed for the out-of-plane and in-plane modes.

Table 1. Comparison between natural frequencies of mode shapes calculated from the FE model and measured experimentally from the racket

Mode Shape	*EMA (Hz)*	*FE (Hz)*	**Difference %**
B1	178	180	1.11
IPB1	205	187	9.62
T1	422	422	0
IPB2	445	382	16.49
B2	509	505	0.79
S1	566	571	0.87
S2	883	891	0.89
S3	906	917	1.19
IPB3	987	737	33.92
B3	1013	1021	0.78
T2	1146	1076	6.50
S4	1184	1204	1.66
S5	1191	1209	1.48
S6	1313	1314	0.08
S7	1440	1421	1.33
S8	1448	1436	0.83
		Mean	4.85

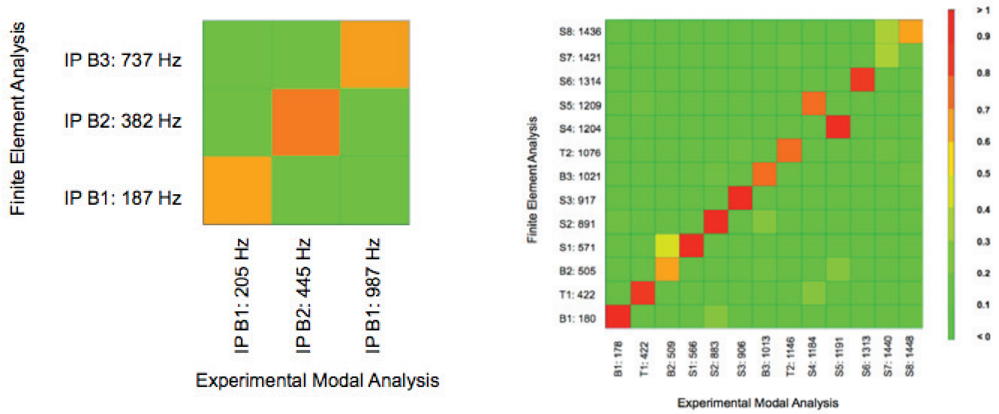

Fig. 4. Modal assurance criterion (MAC) of (a) out-of-plane modes; (b) in-plane modes

5. Discussion and Conclusion

Comparison of the experimental data and the simulation data reveals good agreement between nearly all the out-of-plane natural frequencies (<2 % error) apart from the 2nd torsion mode (T2), which the FE model underestimates by 70 Hz (6.50 %). The correlations of the 3 in-plane modes degrade as the frequency increases, ranging from 9.62 % to 33.92 %.

The modal assurance criterion (MAC) was used to compare the eigenvectors of the mode shapes as a validation tool. The MAC calculated correlation values of above 0.9 for B1, T1, S1, S2, S3 indicating very good correlation. Values above 0.7 where achieved from all other out of planes modes apart from B2, S7 and S8. It is thought that the reason why S7 and S8 where poorly correlated was due to the spatial resolution of the measurement nodes being insufficient to capture the true mode shapes of these higher natural frequencies. Modal switching is a common phenomenon experienced when comparing experimental data and FE data due to inconsistencies within the model. The only modes that have switched places are S4 and S5, which only differ by 5 Hz according to the EMA data. S4 and S5 are also a modal pair with the same number of nodal circles and node lines but slightly different frequencies because the head of the racket is not circular. The MAC values for the three in-plane modes were 0.67, 0.74 and 0.71 respectively. So even though the frequencies of the modes differ greatly, the mode shapes are reasonably correlated. A possible reason for the in-plane modes being less well correlated could be due to the experimental method where the in-plane modes were not excited to the same extent as the out-of-plane modes, leading to noise in the in-plane mode shapes.

This paper has shown that an isotropic model of a tennis racket can exhibit very similar dynamic properties to the associated manufactured racket, especially in the out-of-plane direction. The main differences between the reference model and the FE model were found in the data for the in-plane modes, this could be due to the isotropic nature of the model and might be improved if the model had orthotropic material properties. The validated FE model could be used in the design process with greater confidence that the calculated mechanical properties were representative of the properties that the manufactured racket would have

References

[1] Gu, Y. dong, & Li, J. she. Dynamic simulation of tennis racket and string. International Journal of Sports Science and Engineering, 2007. 1(1), 55–60.

[2] Li, L., Yang, S. H., Hwang, C.-S., & Kim, Y. S. Effects of string tension and impact location on tennis playing. Journal of Mechanical Science and Technology, 2010. 23(11), 2990-2997.

[3] Allen, T., Haake, S., & Goodwill, S.. Comparison of a finite element model of a tennis racket to experimental data. Sports Engineering, 2009, 12(2), 87-98.

[4] Ewins, D. J. Modal testing: theory, practice and application. Research Studies Press. 2000.

[5] Hocknell, A., Mitchell, S., Jones, R., & Rothberg, S. Hollow golf club head modal characteristics: determination and impact applications. Experimental mechanics, 1998. 38(2), 140–146.

[6] Banwell, G., Roberts, J. R., Mohr, S., Rothberg, S. J. Identifying the Modes Excited in a Tennis Racket by a Forehand Drive. IMAC XXX. Florida, USA. 2012.